Dual Dig Level I (2010)

Unless otherwise stated or implied, assume that all graphs are in the usual *xy*-plane.

1. If p is a prime number greater than 2010, which of the following cannot be a prime number?

   A. $p+2$
   B. $p+100$
   C. $p+2009$
   D. $p+2010$

2. The diameter of a circle is d meters. If the circle is enlarged so that the diameter is increased by π meters, what is the increase in the circumference of the circle?

3. What is the greatest odd factor of 6000?

4. Simplify completely: $\dfrac{4x^{-2}-9y^{-2}}{10x^{-1}-15y^{-1}}$

5. Simplify completely: $\sqrt{x\left(\sqrt[3]{x\left(\sqrt[4]{x}\right)}\right)}$

. The digits 1, 2, 3, 4, and 5 are placed in each row and column of a 5x5 matrix so that each column and each row contains only one of each of those digits. Which digit must be placed in the bottom right corner?

_	5	4	_	_
1	3	_	_	_
_	_	5	3	_
2	_	3	1	_
_	_	_	_	??

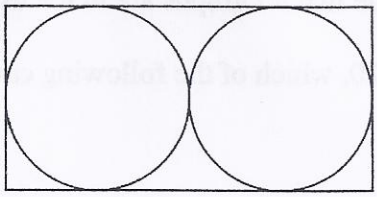
7. Simplify completely: $3+\sqrt{3}+\dfrac{1}{3+\sqrt{3}}+\dfrac{1}{3-\sqrt{3}}$

8. What fraction of the first one million positive integers are perfect squares? Make sure your fraction is simplified.

9. Let $f(x)=\dfrac{x+1}{x-1}$. Evaluate and simplify $f\left(f\left(2010\right)\right)$.

10. Evaluate $999999^2-999998^2$. (This can also be written as: $999{,}999^2-999{,}998^2$.)

11. Two congruent circles lie tangent to each other. Together, they are circumscribed by a rectangle, as shown below (not to scale). If each diagonal of the rectangle is 10 inches, what is the area of the rectangle?



12. If $x+1=10^{2010}$, then what real value of k satisfies $x^2+2x+1=10^k$?

13. Larry and Margee can clean their entire house in 7 hours, while their toddler, Kristen, just by being around, can completely mess it up in only 2 hours. Larry and Margee completely clean their house while Kristen is at her grandparents, and they continue to clean the house once Kristen returns home. From the time Kristen returns home, how long will it be until the house is in complete shambles? (And don't ask why they don't put the kid in a playpen… obviously they're not that smart.)

14. Find all real solutions of: $\log_{(x-4)}(17x-134)=2$

15. Find an equation of the parabola in the usual xy-plane that passes through the points $(0, -4)$, $(1, 5)$, and $(-3, -7)$.

16. A chemist has two solutions of sulfuric acid. The first is half sulfuric acid and half water; the second is 75% sulfuric acid and 25% water. The chemist wishes to use a combination of the two solutions to make 10 liters of a new solution that is two-thirds sulfuric acid and one-third water. How many liters of the half-and-half solution should she use in her new solution?

17. If the diagonal of a cube is d units long, find the <u>surface area</u> of the same cube, in terms of d.

18. Find all real solutions of: $4^{x+1}+16\left(4^{-x}\right)=65$

19. Find the exact <u>area</u> of the region bounded by the graphs of: $\begin{cases} x\sqrt{3}-y=4\sqrt{3} \\ x\sqrt{3}+y=12\sqrt{3} \\ y=0 \end{cases}$

20. Find the remainder when 3^{2010} is divided by 5.